SOLVE IT!

Getting Ready!

The diagram gives information about the populations of two towns. After how many years will the populations be equal? How do you know?

You could make a table to help you model a solution to this problem.

MATHEMATICAL PRACTICES

TOWN A
POPULATION: 3225
Yearly growth:
100 people each year

TOWN B
POPULATION: 3300
Yearly growth:
75 people each year

Let $x =$ # years $3225 + 100x = 3300 + 75x$

To solve multi-step equations, you form a series of simpler equivalent equations. To do this, use the properties of equality, inverse operations, and properties of real numbers. You use the properties until you isolate the variable.

SOLVING EQUATIONS

1. Simplify, if possible
2. If there are variables on both sides, eliminate one
3. Undo + or −
4. Undo • or ÷
5. Check

PROBLEM 1: SOLVING AN EQUATION WITH VARIABLES ON BOTH SIDES
Solve each equation. Check your solutions.

a) $5x + 2 = 2x + 14$
$-2x \qquad -2x$

$3x + 2 = 14$
$-2 \qquad -2$

$\dfrac{3x}{3} = \dfrac{12}{3}$

$\boxed{x = 4}$ ✓

b) $5x - 1 = x + 15$
$-x \qquad -x$

$4x - 1 = 15$
$+1 \qquad +1$

$\dfrac{4x}{4} = \dfrac{16}{4}$

$\boxed{x = 4}$ ✓

c) $-3c - 12 = -5 + c$
$+3c \qquad +3c$

$-12 = -5 + 4c$
$+5 \qquad +5$

$\dfrac{-7}{4} = \dfrac{4c}{4}$

$\boxed{-\dfrac{7}{4} = c}$ ✓

d) $2b + 4 = -18 - 9b$
$+9b \qquad +9b$

$11b + 4 = -18$
$-4 \qquad -4$

$\dfrac{11b}{11} = \dfrac{-22}{11}$

$\boxed{b = -2}$ ✓

e) $7k + 2 - 4k = -10$
$-4k \qquad -4k$

$3k + 2 = -10$
$-2 \qquad -2$

$\dfrac{3k}{3} = \dfrac{-12}{3}$

$\boxed{k = -4}$ ✓

f) $3 + 5q = 9 + 4q$
$-4q \qquad -4q$

$3 + q = 9$
$-3 \qquad -3$

$\boxed{q = 6}$ ✓

g) $8 - 2y = -3y - 2$
$+2y \qquad +2y$

$8 = 5y - 2$
$+2 \qquad +2$

$\dfrac{10}{5} = \dfrac{5y}{5}$

$\boxed{2 = y}$ ✓

h) $n - 24 = 5 - 2n$
$+2n \qquad +2n$

$n - 24 = 5$
$+24 \qquad +24$

$\boxed{n = 29}$

PROBLEM 3: SOLVING AN EQUATION WITH GROUPING SYMBOLS
Solve each equation. Check your solutions.

a) $2(5x - 1) = 3(x + 11)$

$10x - 2 = 3x + 33$
$-3x \qquad -3x$

$7x - 2 = 33$
$+2 \qquad +2$

$\dfrac{7x}{7} = \dfrac{35}{7}$

$\boxed{x = 5}$ ✓

b) $8 - 2(3 + b) = b - 9$

$8 - 6 - 2b = b - 9$

$2 - 2b = b - 9$
$+2b \quad +2b$

$2 = 3b - 9$
$+9 \qquad +9$

$\dfrac{11}{3} = \dfrac{3b}{3}$

$\boxed{\dfrac{11}{3} = b}$ ✓

c) $2(g + 4) - 3g = 1 + 2g$

$2g + 8 - 3g = 1 + 2g$

$-g + 8 = 1 + 2g$
$+g \qquad\qquad + g$

$8 = 1 + 3g$
$-1 \quad -1$

$\dfrac{7}{3} = \dfrac{3g}{3}$

$\boxed{\dfrac{7}{3} = g}$ ✓

d) $5g - 4(-5 + 3g) = 1 - g$

$5g + 20 - 12g = 1 - g$

$-7g + 20 = 1 - g$
$+7g \qquad\qquad +7g$

$20 = 1 + 6g$
$-1 \quad\quad -1$

$\dfrac{19}{6} = \dfrac{6g}{6}$

$\boxed{\dfrac{19}{6} = g}$ ✓

e) $2r - (5 - r) = 13 + 2r$

$2r - 5 + r = 13 + 2r$

$3r - 5 = 13 + 2r$
$-2r \qquad\quad -2r$

$r - 5 = 13$
$+5 \quad +5$

$\boxed{r = 18}$ ✓

f) $4(2y + 1) = 2(y - 13)$

$8y + 4 = 2y - 26$
$-2y \qquad -2y$

$6y + 4 = -26$
$-4 \quad\quad -4$

$\dfrac{6y}{6} = \dfrac{-30}{6}$

$\boxed{y = -5}$ ✓

g) $-8x - (3x + 6) = 4 - x$

$-8x - 3x - 6 = 4 - x$

$-11x - 6 = 4 - x$
$+11x \qquad\quad +11x$

$-6 = 4 + 10x$
$-4 \quad -4$

$\dfrac{-10}{10} = \dfrac{10x}{10}$

$\boxed{-1 = x}$ ✓

h). $14 + 3n = 8n - 3(n - 4)$

$14 + 3n = 8n - 3n + 12$

$14 + 3n = 5n + 12$
$-3n \quad -3n$

$14 = 2n + 12$
$-12 \qquad -12$

$\dfrac{2}{2} = \dfrac{2n}{2}$

$\boxed{1 = n}$ ✓

An equation that is true for every possible value of the variable is an **_identity_**. An equation has no solution if there is no value for the variable that makes the equation true.

PROBLEM 4: SPECIAL CASES—IDENTITIES AND EQUATIONS WITH NO SOLUTION
Solve each equation. If the equation is an identity or has no solution, state it.

a) $10x + 12 = 2(5x + 6)$

$10x + 12 = 10x + 12$
$-10x \qquad -10x$

$12 = 12$ ✓

$\boxed{\text{Identity}}$

$\{\text{all real numbers}\}$

b) $9m - 4 = -3m + 5 + 12m$

$9m - 4 = 5 + 9m$
$-9m \qquad\quad -9m$

$-4 \neq 5$

$\boxed{\text{No Solution}}$

c) $4(3m + 4) = 2(6m + 8)$

$12m + 16 = 12m + 16$

$\boxed{\text{Identity}}$

d) $-6a + 3 = -3(2a - 1)$

$-6a + 3 = -6a + 3$
$+6a \qquad\quad +6a$

$3 = 3$ ✓

$\boxed{\text{Identity}}$

e) $k - 3k = 6k + 5 - 8k$

$-2k = 5 - 2k$
$+2k \qquad +2k$

$0 \neq 5$

$\boxed{\text{No Solution}}$

f) $4 - d = -(d - 4)$

$4 - d = -d + 4$
$+d \quad\quad +d$

$4 = 4$ ✓

$\boxed{\text{Identity}}$

PROBLEM 2: REAL-WORLD PROBLEM SOLVING

a) It takes a graphic designer 1.5 hours to make one page of a Web site. Using new software, the designer could complete each page in 1.25 hours, but it takes 8 hours to learn the software. How many Web pages would the designer have to make in order to save time using the new software?

Let w = web pages

32 Web pages

$$1.5w = 1.25w + 8$$
$$-1.25w \quad -1.25w$$
$$.25w = 8$$
$$\frac{.25w}{.25} = \frac{8}{.25}$$
$$\boxed{w = 32}$$

b) An office manager spent $650 on a new energy-saving copier that will reduce the monthly electric bill for the office from $112 to $88. In how many months will the copier pay for itself?

Let m = months

28 months

$$112m = 88m + 650$$
$$-88m \quad -88m$$
$$\frac{24m}{24} = \frac{650}{24}$$
$$\boxed{m = 27.08\overline{3}}$$

c) A skier is trying to decide whether or not to buy a season ski pass. A daily pass costs $67. A season ski pass costs $350. The skier would have to rent skis with either pass for $25 per day. How many days would the skier have to go skiing in order to make the season pass less expensive than daily passes?

Let d = days

6 days

$$67d + 25d = 350 + 25d$$
$$92d = 350 + 25d$$
$$-25d \qquad -25d$$
$$\frac{67d}{67} = \frac{350}{67}$$
$$\boxed{d = 5.2\ldots}$$

d) A small juice company spends $1200 per day on business expenses plus $1.10 per bottle of juice they make. They charge $2.50 for each bottle of juice they produce. How many bottles of juice must the company sell in one day in order to equal its daily costs?

Let b = bottles

858 bottles

$$1200 + 1.10b = 2.50b$$
$$-1.10b \qquad -1.10b$$
$$\frac{1200}{1.4} = \frac{1.4b}{1.4}$$
$$\boxed{857.14 = b}$$

e) The sum of three consecutive integers is equal to four times the smallest of the integers. Write and solve an equation to find the integers.

Let $n =$ smallest integer : 3

$n + (n+1) + (n+2) = 4n$

$n+1 : 4$

$n+2 : 5$

$3,4,5$

$3n + 3 = 4n$
$-3n \qquad -3n$
$$\boxed{3 = n}$$

f) Shirley is going to have the exterior of her home pained. Tim's Painting charges $250 plus $14 per hour. Colorful Paints charges $22 per hour. How many hours would the job need to take for Tim's Painting to be the better deal?

g) Three times the sum of a number and 4 is 8 less than one-half the number. Write and solve an equation to find the number.

Let $n =$ number

-8

$3(n+4) = 0.5n - 8$

$3n + 12 = 0.5n - 8$
$-0.5n \qquad -0.5n$

$2.5n + 12 = -8$
$\quad -12 \quad -12$

$\dfrac{2.5n}{2.5} = \dfrac{-20}{2.5}$ $\qquad \boxed{n = -8}$

Superstar problem) The sum of two consecutive even integers is equal to 46 less than 8 times the smaller integer. Write and solve an equation to find the integers.

Let $n =$ even integer

$n+2$

Homework

Evens &

7, 23, 25